Deterministic OR Models and Their Formulations

Hakan Özaktaş
Department of Industrial Engineering
Çankaya University

November 11, 2014
What is Operations Research?

- History of OR dates back to World War II:
  - British and American armies employed their best mathematicians to solve some of their military problems.
    - Deployment of radars
    - Management of vehicle convoys
    - Loading bomber planes
    - Minimize damage on bomber planes
  - The original term is the Research of Military Operations.

- Operations Research is the discipline dealing with studies to develop models and/or procedures to aid management in making the correct decisions when confronted with a problem concerning allocation of resources among competing activities.
What is Operations Research?

• Definition by Operational Research Society of the UK:
  – “The attack of modern science on complex problems arising in the direction and management of large systems of men, machines, materials, and money in industry, business, government, and defense. The distinctive approach is to develop a scientific model of the system, incorporating measurement of factors such as chance and risk, with which to predict and compare the outcomes of alternative decisions, strategies, or controls. The purpose is to help management determine its policies and actions scientifically.”
Impact of OR on Industrial Engineering

• After World War II the OR approach has been extensively used to solve Industrial Engineering problems encountered in various fields such as:
  – Inventory Planning
  – Scheduling
  – Vehicle Loading / Vehicle Routing
  – Facilities Location, Layout Planning
  – Logistics and Supply Chain Management
Sample OR Problems

• Determining the number of times the beverage truck should arrive at a restaurant to minimize total purchasing and order costs.

• Finding the optimal number and locations of the stations of a suburban railway route.

• Weekly distribution of jobs to workers in a machine shop to minimize overtime working hours.

• Finding the minimum number of call center operators so that the average waiting time per customer is less than 45 seconds.
The Optimal Product Mix Problem

- Two types of chairs: (1) regular, (2) deluxe
- Net profit per chair:
  - (1) 80 liras per regular chair, (2) 125 liras per deluxe chair
- Which chair should be produced?
  - INSUFFICIENT INFORMATION
- Is there any missing information in this question?
  - YES
- 3 resources needed for these products:
  - Wood
  - Metal
  - Labor
The Optimal Product Mix Problem

- (1) regular chair requirements
  - 2 meters of wood, 3 kg of steel, 2 labor-hours
- (2) deluxe chair requirements
  - 3 meters of wood, 4 kg of steel, 5 labor-hours
- Availability
  - (1) Wood: at most 200 meters per week, (2) Steel: at most 400 kg per week, (3) Labor-hours: at most 250 per week
- Which chair should be produced?
  - NOW WE SHOULD BE ABLE TO ANSWER
The Optimal Product Mix Problem

• Let
  – \( x_1 \): Number of regular chairs to be produced per week
  – \( x_2 \): Number of deluxe chairs to be produced per week

• Net profit function:
  – \( 80x_1 + 125x_2 \)

• Resource constraints
  – \( 2x_1 + 3x_2 \leq 200 \) \{Wood availability per week\}
  – \( 3x_1 + 4x_2 \leq 400 \) \{Steel availability per week\}
  – \( 2x_1 + 5x_2 \leq 250 \) \{Labor availability per week\}

• Which chair should be produced?
  – ONE SHOULD SEEK MAXIMIZING THE NET PROFIT FUNCTION. THE OPTIMAL PRODUCTION PLAN COULD ALSO POSSIBLY BE A MIXTURE OF BOTH CHAIRS.
Problem Formulation

• Decision variables
  – $x_1$: Number of regular chairs to be produced per week
  – $x_2$: Number of deluxe chairs to be produced per week

• Objective function:
  – Maximize $80x_1 + 125x_2$

• Subject to constraints
  – $2x_1 + 3x_2 \leq 200$ \{Wood availability per week\}
  – $3x_1 + 4x_2 \leq 400$ \{Steel availability per week\}
  – $2x_1 + 5x_2 \leq 250$ \{Labor availability per week\}
  – $x_1, x_2$ should be nonnegative and integer variables
Difference between Deterministic and Probabilistic OR Models

• Consider a game in which the outcome of each decision is known in advance
  – We know the price to pay beforehand,
  – It is possible that this outcome is very costly to assess,
  – Or it is possible that the assessment of each outcome would be a very time-consuming task (number of choices can be extremely large),
  – But if we ignore these reservations, what remains in a deterministic model is to choose the ‘best’ course of action.

• Consider another game
  – The outcome of our choices is not known in advance,
  – Hopefully, we can speak of a likelihood of outcomes given a certain choice.
Difference between Deterministic and Probabilistic OR Models

• Is the optimal product mix model deterministic or probabilistic?
  – I know my net profit once I make my decision (the number of chairs from each type to produce).

• Suppose there was uncertainty due to following
  – Unsure of net profit per chair,
  – Uncertainty as to resource requirements,
  – Unsure, if I will ever sell any chairs.

• Real life problems should have a degree of uncertainty

• Why then, deterministic models?
  – Simplicity of deterministic models (when compared to equivalent probabilistic models) make them attractive.
Some Important Milestones of OR

• Game Theory
  – John Von Neumann, Oskar Morgenstern

• Development of Linear Programming Models
  – George Dantzig, Leonid Kantorovich, Tjalling Koopmans
  – Problems such as Product Mix, Resource Allocation, Diet Problem
  – Extensively used since 1940s

• Network Programming Models
  – Transportation Problem
  – Transshipment Problem
  – Assignment Problem and its variations
Some Important Milestones of OR

• Integer Programming Problem
  – Knapsack Problem,
  – Travelling Salesman Problem,
  – Despite the availability of elegant models, solution tools are limited when compared to linear programming tools (heuristics are used whenever optimal algorithms are not helpful).

• Nonlinear Programming Problem
  – History of nonlinear programming goes back to 18\textsuperscript{th} century,
  – Solution tools available for models of much smaller scale when compared to linear programming.
Relevant and Supporting Fields

- Calculus
- Linear Algebra
- Probability Theory
- Statistics and Regression Analysis
- Simulation
- Sensitivity Analysis and Post-Optimal Analysis
The Methodology of Operations Research

• The seven-step methodology of Operations Research can be illustrated by the following activities:
  – Define and formulate the problem,
  – Observe the system and collect data,
  – Formulate a mathematical model of the problem,
  – Verify the model,
  – Select a suitable alternative,
  – Present the results and conclusions to the high-level decision makers,
  – Implement and evaluate recommendations.
Two significant topics relevant to OR Approach

• Model building
  – Representation of real life,
  – Simplification,
  – Proper model construction is the key to success.

• Solution tools / methods based on the model
  – Some methods can be complicated, time-consuming or very costly,
  – Approximation approaches, heuristics.
Models and Modeling

• A model is an abstract representation of a real-world phenomenon.

• Models should represent key properties of the entities which they portray.

• Models can be classified as:
  – Iconic models: which looks like (physically) the object itself.
  – Analog models: represent the property of the object with that of another object.
  – Mathematical models: represent the property of the object by means of equations or mathematical statements.
Models and Modeling

- For most practical purposes mathematical models are used.
- No mathematical model can capture all characteristics, properties and uncertainties of a real-life phenomenon.
- Models are a compromise between accuracy and simplicity.
Common OR Models / Solution Tools

• Linear Programming is a tool for solving optimization problems subject to constraints.
  – All the constraints and the objective function are linear,
  – SIMPLEX algorithm,
  – Specialized software.

• Transportation, Assignment, and Transshipment Problems are special cases of Linear Programming.
  – Although these problems are special cases of linear programming much more efficient tools are available.
Common OR Models / Solution Tools

- Integer Programming is a mathematical programming formulation where the decision variables should be integers. IP models can be linear or nonlinear.
  - Pure Integer Programming problems,
  - Mixed Integer Programming problems,
  - Binary (0-1) Integer Programming problems.
- Nonlinear Programming is also optimization with respect to constraints, however not necessarily linear.
OR Professional Organizations

- ORSA: Operations Research Society of America
- TIMS: The Institute for Management Sciences
- INFORMS since 1995
- EURO: Association of European Operational Research Societies
- YAD: Yönetim Araştırması Derneği
Conclusion

• Although real life examples have many elements of uncertainty, relative simplicity of deterministic models make them very valuable for problem solving.
• For simpler models we can expect more accurate, faster and less costly solution tools when compared to complicated models.

• The following textbooks were referred to prepare this lecture: